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Influence of the temperature state on the damageability due to the creep of claddings of cylindrical fuel elements

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This paper deals with the deformation and damageability of the fuel cladding of nuclear reactors, taking into account the creep and the temperature fields across the thickness. Mathematical models and quantitative estimates for durability of the fuel cladding, obtaining using computer simulations, are presented.
Keywords: damageability; creep; fuel cladding; durability; computer simulation

Ромашов Ю.В., Поволоцький Е.В. **Вплив температурного стану на пошкоджуваність внаслідок повзучості оболонок циліндричних тепловиділяючих елементів.** Наводяться отримані за допомогою комп'ютерного моделювання кількісні оцінки впливу температурних полів на довговічність оболонок тепловиділяючих елементів ядерних реакторів.
Ключові слова: пошкоджуваність; повзучість; оболонка твелу; довговічність; комп'ютерне моделювання

Ромашов Ю.В., Поволоцкий Э.В. **Влияние температурного состояния на повреждаемость вследствие ползучести оболочек цилиндрических тепловыделяющих элементов.** Приводятся полученные с помощью компьютерного моделирования количественные оценки влияния температурных полей на долговечность оболочек тепловыделяющих элементов ядерных реакторов.
Ключевые слова: повреждаемость; ползучесть; оболочка твэла; долговечность; компьютерное моделирование

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1. Introduction

Inhomogeneous temperature fields and creep deformations exist during operation, and it is necessary to estimate the influence of these temperature fields on the damageability due to the creep of the fuel cladding of nuclear reactors.

Really, the temperature in centers of fuel pins is limited about 1973K, but the temperature in the outer boundary of the cladding, contacting with the heat carrier, is about 630K. At the same time, the outer diameter of the cladding is about 10mm in modern nuclear reactors with the ceramic nuclear fuel and the pressurized water as the heat carrier, which are the most widely-used at present

[1]. Thus, the significantly temperature difference about 1343K exists between the places separated by the small distance about 5mm (one half of the diameter), and the width of the cladding occupies about 0,5mm of this distance. The temperature in the inner boundary of the cladding is defined by the heat conduction from the heated fuel pellet across the gaseous filled the gap and next thru the cladding to the heat carrier cooling the reactor core [1].

It is well-known that the temperature in the inner boundary of the cladding can be estimated for the given density of internal heat sources in fuel pellets, for the given heat contact conditions between the pellet and the gas, between the gas and the cladding, considering also given heat transfer conditions between the cladding and the heat carrier [2]. The heat transfer conditions between the outer boundary of the cladding and the moving heat carrier must provide the limited difference of temperatures in the inner and outer boundaries of the cladding due to the strength limiting conditions with the temperature stresses considering; this difference can be about 20-30K for the structural materials, widely using for manufacturing elements of the reactors core. The mode of the heat carrier flow and the state of the outer of the fuel assemblies are can be controlled in the corresponding limits to provide the necessary heat transfer conditions between the outer boundary of the cladding and the heat carrier, although the condition of heat transfer between the fuel cladding and the heat carrier can be different in the each places of the reactor core due to the local properties of the flow and can have significant changes during operation. From other side, it is practically impossible to accurately control during operation the internal state of each one of the fuel elements, especially the density of internal heat sources in the fuel pellets and the width of the gaseous gap inside the fuel element, which can changed due to deformations of the pellet and cladding and significantly defines the heat transfer processes, when the core of modern nuclear reactors is formed by means about 50000 fuel elements [1], [2]. Due to these circumstances, the temperature fields can significantly differ in the cladding of each fuel elements, and it is practically impossible to predict the temperature field in each cladding absolutely accurately. Thus, the temperature fields in the cladding can be significantly different in each of the fuel elements even for the similar heat transfer conditions between the outer boundary of the cladding and the moving heat carrier, i.e. the average temperature in the wall thickness of some cladding can be noticeably greater or lesser relatively the expected value, averaged for the all core of the reactor.

The damages due to the creep deformation growth are the one of the most dangerous for the fuel cladding of nuclear reactors, because they can lead to the leakage of the activated fission products form inner volume of the cladding into the heat carrier, which is not isolated and can fall into the environment [1], [2], [3]. The creep deformation growth is significantly depended on the temperature, and it is necessary to have the estimations for influence of the temperature fields on the damageability due to the creep of the cladding [2], [3], [4], especially when the temperature fields can not be estimated accurately.

2. Mathematical formulation of the problem

The possibilities of experimental approaches are significantly limited for research the processes in the reactor core, because it is very difficult and sometime practically impossible to create artificially the conditions similar existing in the reactor core during operation. Thus, the mathematical modeling and computer simulation are the most effective approaches to research the processes in the reactor core in different operation modes, including the damageability of structural elements and different disasters. Next, we will consider the mathematical models of mechanical behavior under creep deformations and damageability for the fuel cladding with cylindrical shape, which is widely used in modern nuclear reactors [2] and, very possibly, will be used in reactors of next generations.

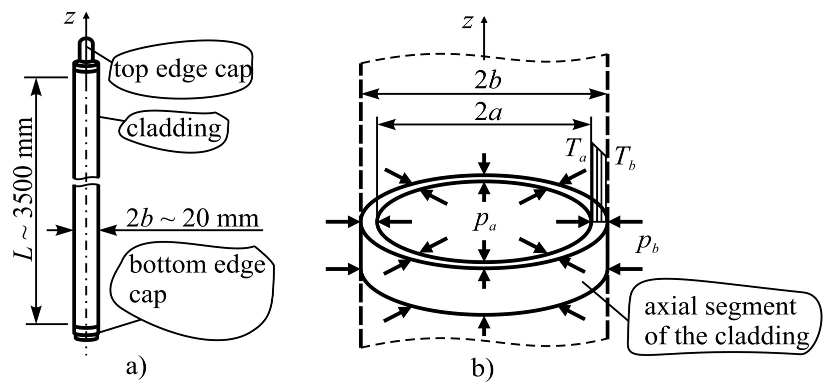
The characteristic length L is significantly greater than the outer diameter b of the fuel element's cladding in modern reactors (pic. 1-a). Due to this circumstance, it is possible to consider the stress-strain state of the cladding only for its axial segment (pic. 1-b), which can be imagined as the thick round cylinder under the small deformations under the well-known conditions of the plane problem [5], [6], taking into account the inner pressure produced by the gases between the pellets and cladding, the outer pressure produced by the moving heat carrier, the temperature field produced by heat conduction thru the wall thickness, as well as creep deformations. Thus, it is necessary to consider the linear kinematic equations for total deformations, and the equilibrium equations, and the linear equations of elastic deforming, taking into account that the total deformations consist of the elastic and creep deformations. It is well-known [5], [6], how to reduce the general view of listed equations, to the particular equations for the plane quasi-static problem for thick-walled round cylinder under the noticed above assumptions. It is not necessary in this article to present the way from listed above general equations to the particular equations, corresponding to the assumptions of the plane problem, but it is necessary to present the differential equations and the boundary conditions of the mathematical model of the stress-strain state of the cladding in the final view used for computer simulations:

$$\begin{aligned}
&-\frac{1}{E}\sigma_r + \frac{\nu}{E}\sigma_\theta + \frac{\partial u}{\partial r} = \alpha(T - T_0) + c_r - \frac{\nu}{E}\sigma_z, \quad a \le r \le b,\\
&\frac{\nu}{E}\sigma_r - \frac{1}{E}\sigma_\theta + \frac{u}{r} = \alpha(T - T_0) + c_\theta - \frac{\nu}{E}\sigma_z, \quad a \le r \le b,\\
&\sigma_z = p_a\frac{a^2}{b^2 - a^2} - p_b\frac{b^2}{b^2 - a^2}, \quad a \le r \le b,\\
&\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_\theta - \sigma_r}{r} = 0, \quad a < r < b,
\end{aligned} \tag{1}$$

$$\sigma_r|_{r=a} = -p_a, \quad \sigma_r|_{r=b} = -p_b, \tag{2}$$

where a, b are the inner and outer radii of the cladding; r is the radial coordinate across the cladding wall thickness; σ_r, σ_θ, σ_z and u are the radial, circumferential, axial stresses and radial displacement; E, ν and α are Young's module, Poisson's ratio and the thermal expansion coefficient of the material used to make the cladding; c_r, c_θ are the radial and circumferential creep deformations; T is the temperature of the cladding; $T_0 = 293K$ is the temperature of the naturally unloaded state of the cladding; p_a and p_b are the inner and the outer pressures, acting on the cladding.

It is possible to find the stresses $\sigma_r = \sigma_r(r)$, $\sigma_\theta = \sigma_\theta(r)$ and displacement $u = u(r)$ fields at the some time $t \geq 0$ by solving the linear boundary-value problem (1), (2) for the given pressures p_a, p_b, for the given temperature field $T = T(r)$ and for the given creep deformations $c_r = c_r(r,t)$, $c_\theta = c_\theta(r,t)$ at this time moment t. The average values of the pressures p_a, p_b are usually known for the given type of nuclear reactors and for modern types of widely-used pressurized-water reactors these values are $p_a \approx 6MPa$ and $p_b \approx 16MPa$ [1], [2], but it is necessary to have the additional equations to find the temperature T and the creep deformations c_r, c_θ.



Pic. 1. Fuel element schematic design (a) and axial segment of it's cladding (b)

The temperature field in the wall of the cladding must be found as the solution of the heat conduction problem for the cylinder, representing the cladding. Next, we will take into account the temperature field, corresponding to the well-known axi-symmetrical stationary heat conduction, which can be represented in the form of boundary-value problem [7]:

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0, \quad a \leq r \leq b, \qquad T|_{r=a} = T_a, \quad T|_{r=b} = T_b, \tag{3}$$

where T_a and T_b are the temperatures on the inner and outer radii of the cladding, and it is evidently that $T_a \geq T_b$, because the heat sources are inside the cladding.

The solution of the heat conduction problem (3) can be easily found by analytical integration of differential equation and analytical finding the necessary constants from the boundary conditions. It is not necessary to show the elementary analytical transformations, leading to the solution of the heat conduction problem (3), but it is necessary to present final well-known result for the temperature field in the cylindrical wall:

$$T(r) = T_b - (T_b - T_a)\frac{\ln(r/b)}{\ln(a/b)}, \quad a \le r \le b. \tag{4}$$

where T_a and T_b are the temperatures on the inner and outer radii of the cladding; it is evidently that $T_a \ge T_b$ because the heat sources are inside the cladding.

The result (4) for the temperature field will be used next in the computer simulations of the cladding.

To find the creep deformations and the damageabilities in the structural material of the cladding, it is necessary to propose the corresponding mathematical model. This model can be represented in the form of differential equations with initial conditions, which for wide classes of the creep deformations and the damageabilities, which are existed in different structural materials, can be represented by using the Cachanov-Rabotnov scalar damage parameter in the next well-known [6], [8] view:

$$\begin{aligned} &\frac{\partial c_r}{\partial t} = \frac{3}{2}\frac{f_c(c_{eq},\omega,\sigma_{eq};T)}{\sigma_{eq}}(\frac{2}{3}\sigma_r - \frac{1}{3}\sigma_\theta - \frac{1}{3}\sigma_z),\\ &\frac{\partial c_\theta}{\partial t} = \frac{3}{2}\frac{f_c(c_{eq},\omega,\sigma_{eq};T)}{\sigma_{eq}}(\frac{2}{3}\sigma_\theta - \frac{1}{3}\sigma_r - \frac{1}{3}\sigma_z),\\ &\frac{\partial \omega}{\partial t} = f_\omega(c_{eq},\omega,\sigma_{eq};T), \end{aligned} \tag{5}$$

$$c_r(r,0) = 0, c_\theta(r,0) = 0, \omega(r,0) = 0, \quad a \le r \le b, \tag{6}$$

where $\omega = \omega(r,t)$ is the Cachanov-Rabotnov scalar damage parameter; $f_c(c_{eq},\omega,\sigma_{eq};T)$ and $f_\omega(c_{eq},\omega,\sigma_{eq};T)$ are the velocities of the creep equivalent deformation and the damage parameter; $c_{eq} = c_{eq}(c_r,c_\theta)$ and $\sigma_{eq} = \sigma_{eq}(\sigma_r,\sigma_\theta)$ are the equivalent creep deformation and the equivalent stress necessary to equivalence the uni-axial and multi-axial stress-strain states.

The effect of the temperature field influence on the damageability of cladding can be estimated by using the mathematical model (1)-(6) due to the presence of the temperature field (4) in the mathematical model (1), (2) of the stress-strain state, and in the kinetic equations (5) for the creep deformations and the damage parameter. The velocities $f_c(c_{eq},\omega,\sigma_{eq};T)$ and $f_\omega(c_{eq},\omega,\sigma_{eq};T)$, presenting in creep-damage kinetic equations (5) must be obtained using the experimental data about the rupture of the specimens of materials under the uni-axial tensile in the creep conditions [6], [8], and it is relatively easy to obtain the velocities f_c and f_ω for given temperatures only. Next, we will consider the cladding made from the the zirconium-based alloys which are widely used to make the cladding in modern widely-used reactors [1], [2]. For zirconium-based alloys we will use the creep-damage velocities in the form:

$$\begin{gathered} f_c(c_{eq}, \omega, \sigma_{eq}; T) = 0,25 f_\omega(c_{eq}, \omega, \sigma_{eq}; T), \\ f_\omega(c_{eq}, \omega, \sigma_{eq}; T) = B_1(T) \left(\frac{\sigma_{eq}}{1-\omega} \right)^{k_1(T)} \left(1 - 2^{-\exp\left(100^{\sigma_{eq} - S(T)}\right)} \right) + \\ + B_2(T) \left(\frac{\sigma_{eq}}{1-\omega} \right)^{k_2(T)} 2^{-\exp\left(100^{\sigma_{eq} - S(T)}\right)}, \end{gathered} \tag{7}$$

where $B_1(T)$, $k_1(T)$, $B_2(T)$, $k_2(T)$ and $S(T)$ are the parameters defined for the given temperature T; $2^{-\exp\left(100^{\sigma_{eq}-S(T)}\right)}$ is the approximation of the step function [9]; $\sigma_{eq} = \sqrt{(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2}/\sqrt{2}$.

First relation (7) form is in the consequence of zirconium-based alloys properties that the rupture creep deformation is not depended on the stress and equals approximately 0,25 [10]. Two items, introduced in the second relation (7), are necessary to take into account the differences in the tilts angles of two characteristic straight sections, typically existing when logarithmic coordinates are used to represent the depending between the rupture time and stress under the creep for the given temperature and existing also for the zirconium-based alloys [10]. For the available experimental data, consisting of the stresses and corresponding to them the times of the rupture times for the given temperature, as, for example, in [10], it is possible to find the parameters $B_1(T)$, $k_1(T)$, $B_2(T)$, $k_2(T)$ and $S(T)$ values, corresponding to that given temperature, by using the least squares. It is impossible to represent the parameters $B_1(T)$, $k_1(T)$, $B_2(T)$, $k_2(T)$ and $S(T)$values, corresponding to the given temperature, in the form of finite formulas, but it is only possible to propose the algorithms, which allow to obtain these parameters values; it is not suitable do discuss these algorithms in this article, but it is necessary to note, that these algorithms are reduced to the least squares applications for different combinations of points from the available experimental data to find the optimal matching to these data. Thus, it is possible to find the parameters $B_1(T)$, $k_1(T)$, $B_2(T)$, $k_2(T)$ and $S(T)$ values in equations (7) for same given temperature, using the necessary available corresponding experimental data, but are no possibilities to find these parameters values for some required temperatures without corresponding experimental data. In this situation it seems naturally to build the extrapolation of available experimental data such that to have the data necessary to find the parameters $B_1(T)$, $k_1(T)$, $B_2(T)$, $k_2(T)$ and $S(T)$ values for some required temperature when are no experimental data for this required temperature. This approach naturally requires the extrapolation of the experimental data about dependencies between the rupture time and the stress, available for the given temperatures, to the required temperatures, and it is naturally to use the long-term strength extrapolation based on the well-known [11] Larson-Miller parameter

$$\sigma_{eq} = A + B P_{LM}, \quad P_{LM} = T\left(C + \lg(t^*)\right), \tag{8}$$

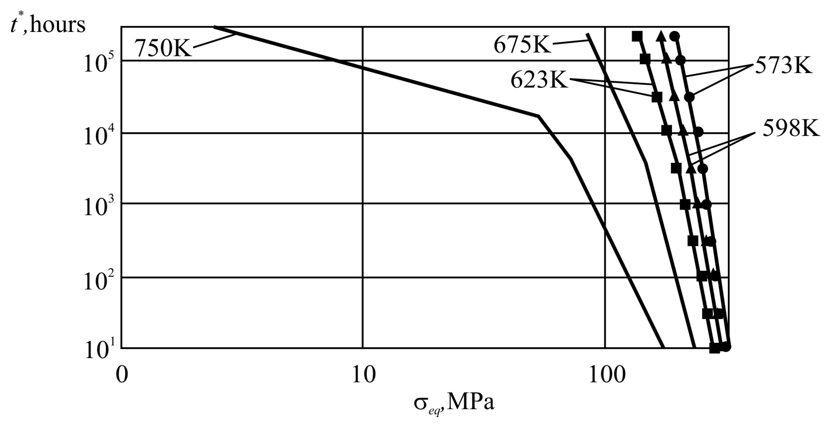
where $A \cong 827,516$, $B \cong -518,932$ and $C \cong 15,895$ are the parameters defined using the primary data about rupture of the Zr-Nb alloy under the creep conditions [10] for the creep in MPa, time in hours and temperature in K; P_{LM} is Larson-Miller parameter; t^* is the rapture time under the creep conditions.

In the case of absence of the finite formulas and clear denoted algorithms for the evaluation the parameters $B_1(T)$, $k_1(T)$, $B_2(T)$, $k_2(T)$ and $S(T)$ from relations (7) as well as of the material constants from the Larson-Miller parameter (8) it is necessary to substantiate using the presented in the form (6)-(8) creep law. For this substantiation it is possible to estimate the rupture times using the creep law (6)-(8), the last equation (5) and the initial condition (6) for the given constant stresses and temperatures:

$$t^* = \int_0^1 \frac{d\omega}{f_\omega(\omega, \sigma_{eq}; T)}, \tag{9}$$

where is took in consideration that the $f_\omega(c_{eq}, \omega, \sigma_{eq}; T)$ is not depended on c_{eq} in the particular case (7) and is reduced to $f_\omega(\omega, \sigma_{eq}; T)$.

Using the formula (9), it is possible to estimate the rupture times, corresponding for the given stresses and temperatures, including the available data used to find the parameters in relations (7), (8), that gives the opportunities to estimate the differences. The results, obtaining using the formula (9), last relation (7) and relation (8), show us (pic. 2) that the relations (7), (8) allow us to predict fairly accurate the rupture time under the creep conditions for wide choice for values of the stress and the temperature; the curves in pic. 2, corresponding to the temperatures $T = 675K$ and $T = 750K$, are obtained only due to the extrapolations using Larson-Miller parameter. It is obviously, that the errors in relations (7), (8) are defined first of all by errors of the Larson-Miller extrapolations, which are allowed for the engineering applications [11]. Thus, the results, presenting in pic. 2, give some substantiation for the relations (7), (8), defining the velocities of creep deformations and damage parameter.



Pic. 2. Long-term strength data (markers) and computed results (curves) obtained using proposed creep-damage law equations for different temperatures

Solving the equations (1), (5) with the boundary conditions (2) and the initial conditions (6) taking into account the relations (7), (8) allows to find the stress-strain state, the creep deformations and the Cachanov-Rabotnov damage parameter and, as the result, the time t^* before the rupture moment of the cladding, which can be defined by using the condition

$$t^*: \quad \omega(r, t^*) = 1 \quad \forall r, a \le r \le b. \tag{10}$$

Thus, the mathematical model of the cladding deformation and damage accumulation under the inner and outer pressures and the temperature field are proposed in the form (1), (2), (4)-(8) of the system of boundary-value and initial value problems.

3. Approaches for the numerical computer simulations

It is necessary to execute the computer simulations of the cladding damaging for the different temperatures to research the influence of the temperature state on the damageability due to the creep of the cladding. Such computer simulations require to numerically solve the differential equations (1), (5) with the boundary conditions (2) and initial conditions (6) taking into account the temperature field (4) and the relations (7), (8). Next, the semi-discretization method also all-known as the method of lines will be used to solve the differential equations (1), (5). To realize this approach we will use the spatial grid with number n of the nodes with coordinates

$$r_k = a + (k-1)\Delta r, \quad k = 1, 2, ..., n, \tag{11}$$

where $\Delta r = \dfrac{b-a}{n-1}$ is the step of the spatial grid.

We will exclude the spatial derivatives from equations (1) using the finite differences formulas. Thus, the all-known [12] scheme of finite differences method applied to the boundary-value problem (1), (2) lead us to the algebraic relation

$$\mathbf{A}_n^{(1)} \mathbf{u}_n^{(1)} = \mathbf{f}_n^{(1)} + \mathbf{A}_n^{(2)} \mathbf{u}_n^{(2)}, \tag{12}$$

where $\mathbf{u}_n^{(1)}$ is the vector consisting the nodal values of the σ_r, σ_θ and u in the spatial grid nodes (11); $\mathbf{u}_n^{(2)}$ is the vector consisting the nodal values of the c_r, c_θ and ω in the spatial grid nodes (11); $\mathbf{A}_n^{(1)}$, $\mathbf{A}_n^{(2)}$ and $\mathbf{f}_n^{(1)}$ are the matrices and the vector obtained as the results of the excluding of the spatial derivatives using the finite differences formulas.

We can consider the initial-value problem (5), (6) in the grid nodes (11):

$$\frac{\partial \mathbf{u}_n^{(2)}}{\partial t} = \mathbf{f}_n^{(2)} \left(\mathbf{u}_n^{(2)}; \mathbf{u}_n^{(1)} \right), \quad \mathbf{u}_n^{(2)}(0) = \mathbf{0}, \tag{13}$$

where $\mathbf{f}_n^{(2)}$ is the vector corresponding to the velocities of the creep deformations and the damage parameter from the equations (5) obtained in the grid nodes (11).

The relation (12) allows us to exclude the vector $\mathbf{u}_n^{(1)}$ from the equation (13) and as a result leads us to Cauchy problem in the canonical form

$$\frac{\partial \mathbf{u}_n^{(2)}}{\partial t} = \mathbf{f}_n^{(2)}\left(\mathbf{u}_n^{(2)}\right), \quad \mathbf{u}_n^{(2)}(0) = \mathbf{0}, \tag{14}$$

where $\mathbf{f}_n^{(2)}\left(\mathbf{u}_n^{(2)}\right) = \mathbf{f}_n^{(2)}\left(\mathbf{u}_n^{(2)}\right)$; $\mathbf{u}_n^{(1)} = \left(\mathbf{A}_n^{(1)}\right)^{-1}\left(\mathbf{f}_n^{(1)} + \mathbf{A}_n^{(2)}\mathbf{u}_n^{(2)}\right)$.

We will use Merson method [13] to solve the initial-value problem (12), because this method gives us the effective way to define the integrating step on time corresponding to increasing velocities of the creep deformations and the damage accumulations on the third stage [14]. Thus, we have the effective scheme (11)-(14) to numerical solve the equations (1), (5) necessary for the computer simulations of the cladding.

4. Results of the numerical computer simulations

The aim of computer simulations is to obtain the quantity estimations about the temperature influence on the damageability of the zirconium cladding with cylindrical shape widely used in modern reactors. To reach this aim we will consider the computer simulations based on the mathematical model (1)-(8) and numerical solution scheme (11)-(14) for the zirconium cladding with the parameters typical for modern nuclear reactors used the pressurized light water as the heat carrier:

$$\begin{aligned} &a = 3,855mm, \quad b = 4,55mm, \\ &p_a = 6MPa, \quad p_b = 16MPa, \quad T_b = T_a - 25K. \end{aligned}$$

We will consider the computer simulations for the several values of the inner temperature T_a:

$$\begin{aligned} &T_a = 625K, \quad T_a = 650K, \quad T_a = 675K, \\ &T_a = 700K, \quad T_a = 725K, \quad T_a = 750K. \end{aligned}$$

Results of computer simulations depends on the number n of nodes in the grid (10) and it is necessary to substantiate the number n of nodes necessary to obtain the solutions with required errors of the approximation. The simplest practical way to substantiate number n of the nodes is to make the series of computer simulations with different numbers of nodes to compare each other the results of these simulations. It is naturally to use first of all the rupture time (10) of the cladding as the integrated characteristic of the numerical solutions for the differential equations (1), (5). The results of simulations with different numbers

n for the temperature $T_a = 750K$ lead to next results:

$$\begin{array}{ll}
n = 12, & t^* = 1.91462321695327759 \cdot 10^4 hours, \\
n = 22, & t^* = 1.92791736355613031 \cdot 10^4 hours, \\
n = 32, & t^* = 1.97832680938720703 \cdot 10^4 hours, \\
n = 42, & t^* = 2.06514756988187117 \cdot 10^4 hours, \\
n = 52, & t^* = 2.07993760411059251 \cdot 10^4 hours, \\
n = 62, & t^* = 2.03998893927619300 \cdot 10^4 hours, \\
n = 72, & t^* = 2.09723706951267550 \cdot 10^4 hours, \\
n = 82, & t^* = 2.04467821414862175 \cdot 10^4 hours, \\
n = 92, & t^* = 2.09752126695804298 \cdot 10^4 hours, \\
n = 102, & t^* = 2.10205232226189223 \cdot 10^4 hours, \\
n = 112, & t^* = 2.06086735430503826 \cdot 10^4 hours.
\end{array}$$

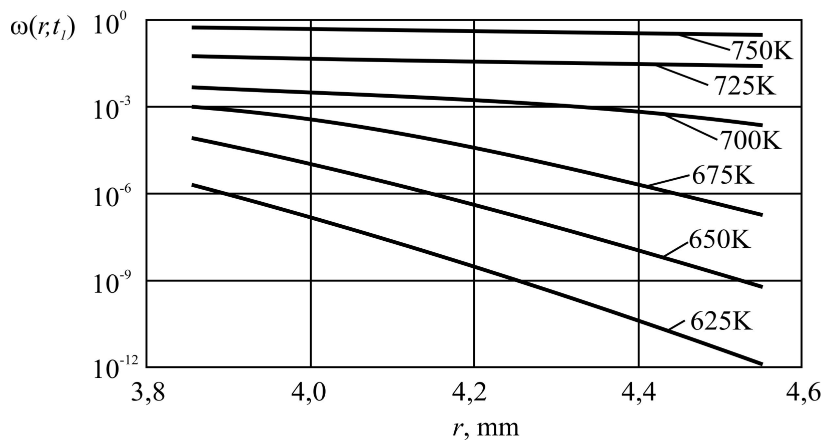
The analysis of the results of the computer simulations obtained for the different number n of the nodes shows that the proposed numerical scheme (11)-(14) with the relatively low numbers n of nodes allows to obtain the numerical solution of the problem (1)-(8) with errors of the approximations available for engineering applications, and number $n = 52$ is sufficiently for computer simulations of the cladding.

The damage parameter $\omega(r,t)$ can be used as the measure of the damageability due to its properties that the value $\omega = 0$ corresponds to the non-damaged state and the value $\omega = 1$ corresponds to the state of rupture. To estimate the temperature influence on the damageability of the cladding it is necessary to present the fields of the damage parameter obtained for different values of the temperature T_a at the some given time moment $t = t_1$; next we use the value t_1 corresponding two years of operation:

$$t_1 = 17520 hours.$$

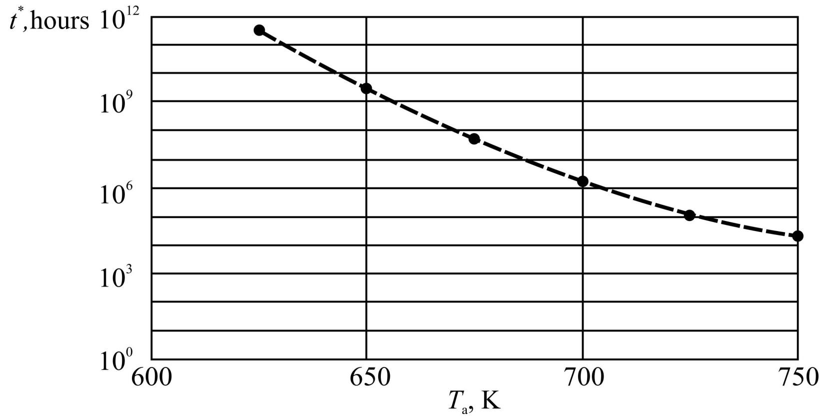
Obtained results showing in the pic.3 give us the quantities estimation of the temperature significant influence on the damageability of the cladding made from the zirconium based alloys. The damageability of the cladding not leads to the rupture during two-years operation even for the temperature $T_a = 750K$. At the same time, possibilities of the operation of the cladding are limited by the leakages of the fission's gaseous products also [1, 2] which can exist without the rupture of the cladding and can be realized by the diffusion mechanisms for example. It is naturally that the damageability of the cladding can accelerate the diffusion mechanisms and increase the leakage of the fission's gaseous products, such that the cladding damaged even without the rupture can not be operated further.

The core of nuclear reactors includes more than 50000 fuel elements and the normal operation admits the rupture not more than 10 fuel elements [2]. It is generally recognized that the ruptures of the cladding are the result of the contact interacting between the fuel pellets and the cladding [2]. This interaction is not fully researched now, but it is obviously that the contact interaction between the pellet and the cladding leads to the increasing the temperature inside the cladding and it is necessary to have the quantity estimations of the temperature influence of the rupture time of the cladding. The computer simulations of the cladding for the different values of the temperature T_a with the relation (10) give us the opportunities to estimate the corresponding rupture times of the cladding that it is leads us to the dependence between the rupture time t^* and the inner temperature T_a, which is presented in pic. 4. It is can be saw (pic. 4) that for the cladding, making from the zirconium-based alloys, the rupture time has a widely changing for the inner temperatures $625K \leq T_a \leq 750K$, corresponding to the parameters of the modern pressurized water reactors. Taking into account that the values $625K \leq T_a \leq 750K$ of the temperature in the inner surface of the cladding are possible for the some particular cladding, the result (pic. 4) shows that the rupture of the some cladding are possible for modern and future generations of the reactors, which use the pressurized light water as a heat carrier and use the zirconium-based alloys as the structural materials of the cladding. With accounting of this circumstances, it is necessary to design the core of reactors with the some given probability of the rupture of the cladding, considering that the ruptures of the cladding for not more than 10 from more than 50000 fuel elements are allowed during required time of operating [1, 2].



Pic. 3. Fields of the damage parameter in the thickness of the cladding obtained in the time moment $t = t_1$ for different values of the temperature T_a

It seems, that the significant rupture time of the cladding, corresponding to the expected value $T_a \approx 650K$ of the temperature in the inner surface during the normal operation, provides the absence of ruptures of the cladding during 4 years operation for modern reactors normally and the duration of the operation of the modern nuclear fuel can be increased significantly. Concerning with this circumstance, it is necessary to note, that the rupture times, presented in pic. 4, are obtained by using the conditions (10), which corresponds to the full mechanical fracture of the cladding with forming the visually observed ruptures. At the same time, the failure of the fuel element is defined by increasing the radioactivity of the heat carrier, which it is possible more early than the mechanical fracture due to the leakages thru the micro-defects in the cladding, forming during operating due to damageabilities including.



Pic. 4. Influence of the inner temperature on the rupture time of the cladding

5. Conclusions

It is evidently from the fundamental knowledge and qualitative assessments in solids mechanics, that the temperature fields have certain influence on the damageability and on the permissible life time of the cladding, but the importance of that influence can be estimated using the only corresponding quantitative data in each from the particular cases. These quantitative data can be obtained using only the computer simulations, because it is difficult to realize the testing conditions similar to the conditions into reactor core, and it is difficult to estimate the damageability in the structural materials using instrumental measures. The computer simulations of the cladding necessary to estimate the influence of the temperature on the damageability must define the changing of the state of the cladding during the operation, corresponding to their modes and conditions, and it is necessary to have the numerical scheme allows to solve the mathematical problem, which will be generated by that mathematical model. The quality of the quantitative data, obtained using the computer simulations, will be defined by the opportunities of the used mathematical models first of all.

The mathematical model of the cladding stress-strain state and damaging due to the inner and outer pressures in the temperature field under the creep conditions was proposed to obtain the quantitative estimation for the influence of the temperature fields on the damageability of the cladding. That mathematical model was presented as the system of the some boundary-value problem and the some initial-value problem, what it allows to estimate changing of the stress-strain states and its influence on changing of the creep deformations and damage parameter. But, it is necessary to note, that the proposed mathematical model has significant limitations in the considering deformations of the cladding and allows to only estimate the axial-symmetric stress-strain state of the cladding, but, at the same time, other deformations, including bending of the cladding, are possible during operation. It is necessary further to have the mathematical models with taking into account the different deformations, especially including bending or 3-D deformations, to more precisely estimate the state of the cladding. The rupture of the cladding is the consequence the result of only the thermal creep deformations and the damages due to the thermal creep in the proposed mathematical model of the cladding, and it is requires the significantly caution to the obtained results, because the different processes lead to rupture at the same time. It is necessary to consider the corrosion, the radiation damaging and creep, and the another damaging processes leading to the rupture to the more precisely estimate the state of the cladding. But, it is sufficiently to consider the only axial-symmetric deformations and the thermal creep to obtain the approximately quantitative estimations of the influence of the temperature field on the damageability of the cladding what it is corresponds to the purpose due to significant influence the temperature on the creep.

The numerical scheme based on the semi-discretization method, also well-known as the method of lines, was proposed to numerically solve the system of the some boundary-value problem and the some initial-value problem, presenting the proposed mathematical model of the cladding. That approach leads to the resolved initial-value problem, which can be solved using the well-known numerical methods. The experience of the computer simulations shows, that the proposed numerical scheme allows to obtain quickly the numerical solutions with the necessary accuracy, and it is possible to recommend to use that numerical scheme in further researches.

It is proposed to use the scalar damage parameter obtained for the some interesting given time and the rupture time as the damageability measures of the cladding. It is shown, that the fields of the damage parameter are noticeably nonuniform in the wall of the cladding. The number values of the damage parameter, testified the absence of the rupture of the cladding for the average temperatures expected for the normal operation, but the computer simulations allow to illustrate the constantly accumulation of the damageabilities in the material. The significant decreasing of the rupture time of the cladding with increasing before 750K of the temperature evidences that the rupture of the some cladding in reactor's core is possible due to the extremal operating conditions, when the difference between the inner and the outer temperatures of the fuel element is about 1350K, and this temperature's difference is realized on 5 mm of the fuel element's transverse dimension due to the heat conductivity and the heat transfer mechanisms. It is assumed, that at this significant temperature differences the temperature on the inner boundary of the cladding is about 650K, and it is shown, using the computer simulations, that the time before rupture of the cladding under the creep conditions at that temperature is practically unlimited. At the same time, the temperature on the inner boundary of the cladding is the result of the heat conduction and the heat transfers in the spatial domain inside the cladding with the size about 5 mm between the temperature about

1400K and 340K and that result is significantly depended from the condition of the heat conduction and the heat transfer. It is possible, that the heat transfer's and the heat conduction's conditions inside of the some fuel element may be significantly differ from the average conditions in the core, consisting of more than 50000 fuel elements, such so in the some of fuel element the temperature on the inner boundary is more than 650K, corresponding to the average conditions and can be 700K or 750 K actually. Due to these circumstances, it is actually necessary to design the cladding of fuel elements with the some given probability of the rupture of the cladding, considering that the ruptures of the cladding for not more than 10 from more than 50000 fuel elements are allowed during operating. Thus, the probability of the rupture is necessary to substantiate quality of the fuel elements and their cladding's design solutions, that must be developed in further researches.

Obtained results for the time before rupture of the cladding, corresponding to the expected value $T_a \approx 650K$ of the temperature in the inner surface during the normal operation, show that the mechanical rupture of the cladding due to the creep is impossible during 4 years operating. This is agreed with experience of exploitation of the fuel elements in industrial nuclear reactors, that failures of the fuel elements occur due to the leakages of the fission products outside the volume of fuel elements, which is defined by increasing the radioactivity of the heat carrier without the visually observed ruptures of the cladding. It is naturally to assume, that the depressurization of the fuel elements is the result of damageabilities of the cladding, including due to the creep. Thus, to estimate the the reliability indexes of the fuel elements it is necessary to propose the mathematical models of the leakages outside the fuel elements, considering with the damageabilities and creep deformations of the cladding.

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Ромашов Ю.В., Поволоцький Е.В. **Вплив температурного стану на пошкоджуваність внаслідок повзучості оболонок циліндричних тепловиділяючих елементів.** Наводяться отримані за допомогою комп'ютерного моделювання кількісні оцінки впливу температурних полів на довговічність оболонок тепловиділяючих елементів ядерних реакторів. Комп'ютерне моделювання здійснено за допомогою математичної моделі деформування та руйнування оболонки твелу внаслідок повзучості під дією тисків осколків ділення та теплоносія в неоднорідному температурному полі, що встановлюється при експлуатації уздовж товщини оболонки. Математична модель представлена у вигляді крайової задачі, яка визначає напружено-деформований стан оболонки з урахуванням деформацій повзучості на основі відомих концепцій механіки деформівного твердого тіла, та початкової задачі, яка визначає розвиток у часі деформацій повзучості та скалярного параметру пошкоджуваності при заданих напруженнях та температурі. Числові параметри, що характеризують конструкційний матеріал у рівняннях для деформацій повзучості та параметру пошкоджуваності, для заданої температури визначалися на основі даних, що отримані шляхом екстраполяції відомих даних, відповідних обмеженій кількості значень температури та напруження, із використанням параметру Ларсона-Мілера. Для розв'язування диференціальних рівнянь, що представляють математичну модель деформування та руйнування оболонки твелу, використано метод напівдискретизації. Результати розрахунків свідчать, що оболонки твелів мають достатньо високу довговічність при температурах, які відповідають середнім температурам, що очікуються в активній зоні реактору. В той же час, розрахунки свідчать, що довговічність оболонок твелів суттєво зменшується до небезпечного рівня при підвищених температурах, які є цілком можливими через локальні відхилення процесів теплообміну в активній зоні реактору. Це є дуже суттєвим, оскільки у використаної математичної моделі граничний стан оболонки твелу, що обмежує можливості її експлуатації, відповідає повному механічному руйнуванню оболонки, хоча насправді експлуатація твелів обмежується більш жорсткими умовами щодо рівня герметичності оболонки, який може не забезпечуватися навіть й для незруйнованої цілком оболонки.

Ключові слова: пошкоджуваність; повзучість; оболонка твелу; довговічність; комп'ютерне моделювання

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