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Continual distribution with screw modes

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Explicit approximate solution of the Boltzmann equation for the hard-sphere model are built. It has the kind of continual distribution in the case of local Maxwellians of special form describing the screw-shaped stationary equilibrium states of a gas. Some limited cases, in which this distribution minimized the uniform-integral remainder between the sides of this equation are obtained.
Keywords: hard spheres, Boltzmann equation, Maxwellian, screws, uniform-integral remainder, continual distribution.

Гордевський В. Д., Сазонова О. С. **Континуальний розподіл з гвинтовими модами.** Побудовано явний наближений розв'язок нелінійного рівняння Больцмана для моделі твердих куль. Він має вид континуального розподілу у випадку локальних максвеліанів, що описують стаціонарні рівноважні стани газу, подібні гвинтам. Здобуто деякі граничні випадки, в яких цей розподіл мінімізує рівномірно-інтегральний відхил між частинами рівняння.
Ключові слова: тверді кулі, рівняння Больцмана, максвеліан, гвинти, рівномірно-інтегральний відхил, континуальний розподіл.

Гордевский В. Д., Сазонова Е. С. **Континуальное распределение с винтовыми модами.** Построено явное приближенное решение нелинейного уравнения Больцмана для модели твердых сфер. Оно имеет вид континуального распределения в случае локальных максвеллианов, описывающих винтообразные стационарные равновесные состояния газа. Получены некоторые предельные случаи, в которых это распределение минимизирует равномерно-интегральную невязку между частями уравнения.
Ключевые слова: твердые сферы, уравнение Больцмана, максвеллиан, винты, равномерно-интегральная невязка, континуальное распределение.
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1. Introduction

The interaction between flows of a gas of hard spheres is described by the kinetic integro-differential Boltzmann equation [1]–[3]:

$$D(f) = Q(f,f), \tag{1}$$

$$D(f) = \frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x}, \tag{2}$$

$$\begin{aligned} Q(f,f) = \frac{d^2}{2}\int\limits_{\mathbb{R}^3} dv_1 \int\limits_{\Sigma} d\alpha |(v-v_1,\alpha)|[f(t,v_1',x)f(t,v',x)- \\ - f(t,v_1,x)f(t,v,x)], \end{aligned} \tag{3}$$

where $f(t,v,x)$ is the distribution to be found, that describes the number of particles that in the moment of time t have velocity v and are in the point of space x, $\partial f/\partial x$ is its spatial gradient, $t \in R^1$ is the time, $x = (x^1,x^2,x^3) \in R^3$ and $v = (v^1,v^2,v^3) \in R^3$ are the molecule coordinate and the velocity, $d > 0$ is its diameter, v and v_1 are the molecule velocities before the collision, $\alpha \in \Sigma$, Σ is the unit sphere in R^3. The molecule velocities after the collision are defined by the formulae

$$v' = v - \alpha(v-v_1,\alpha), \quad v_1' = v + \alpha(v-v_1,\alpha), \tag{4}$$

The well-known exact solutions of (1) – (4) are the global and local Maxwellians [1]-[3]. Some other exact solutions were obtained only for the case of Maxwellian molecules and for some of its generalizations [4]–[6].

That is why the question of the search of explicit approximate solutions of this kinetic integro-differential equation and satisfying it with arbitrary accuracy was occured.

Then bimodal distributions including both global and local Maxwellians of different particular kinds describing screw-shaped [7], [8], tornado-like [9], [10] and other equilibrium states of a gas were studied [11].

Then a new approach to the search for explicit approximate solutions of the Boltzmann equation was proposed in the paper [12], namely the continual kind of distribution function. It was supposed, that mass velocity of the global Maxwellian does not take discrete values but becomes an arbitrary parameter taking any values in $\mathbb{R}^3$.

Attempts to transfer the results of [12] and other works in the case of local Maxwellians of the most general form have not been successful due to occur at the same time significant difficulties.

The objective of this paper is to study the behavior of the continual distribution involving local Maxwellians of a special form that describe the screw-shaped stationary equilibrium states of a gas (in short-screws or spirals) [7, 8, 14]. Every Maxwellian of this type is defined by the formula

$$M(v,u,x) = \rho_0 e^{\beta\omega^2 r^2}\left(\frac{\beta}{\pi}\right)^{\frac{3}{2}} e^{-\beta(v-u-[\omega\times x])^2}. \tag{5}$$

Physically, distribution (5) corresponds to the situation when the gas has an inverse temperature $\beta = \frac{1}{2T}$, where $T = \frac{1}{3\rho} \int\limits_{\mathbb{R}^3} (v-u)^2 f dv$ and rotates in whole as a solid body with the angular velocity $\omega \in R^3$ around its axis on which the point $x_0 \in R^3$ lies,

$$x_0 = \frac{[\omega \times u]}{\omega^2}, \tag{6}$$

The square of this distance from the axis of rotation is

$$r^2 = \frac{1}{\omega^2}[\omega \times (x - x_0)]^2 \tag{7}$$

and the density of the gas has the form:

$$\rho = \rho_0 e^{\beta \omega^2 r^2} \tag{8}$$

(ρ_0 is the density of the axis, that is $r = 0$), $u \in R^3$ is the arbitrary parameter (linear mass velocity for x), for which $x||\omega$, and $u + [\omega \times x]$ is the mass velocity in the arbitrary point x. The distribution (5) gives not only a rotation, but also a translational movement along the axis with the linear velocity

$$\frac{(\omega, u)}{\omega^2}\omega,$$

Thus, it really describes a spiral movement of the gas in general, moreover, this distribution is stationary (independent of t), but inhomogeneous.

We will consider the continual distribution [12] such form as:

$$f = \int\limits_{\mathbb{R}^3} \varphi(t, x, u) M(v, u, x) du, \tag{9}$$

It is assumed, that the coefficient function $\varphi(t, x, u)$ is non-negative and belong to $C^1(\mathbb{R}^7)$. It is required to find functions $\varphi(t, x, u)$ and the behavior of all parameters such that the uniform-integral remainder [13]

$$\Delta = \sup_{(t,x) \in \mathbb{R}^4} \int\limits_{\mathbb{R}^3} |D(f) - Q(f, f)| dv, \tag{10}$$

tends to zero.

In the section 2 asymptotical expressions for some upper estimations of remainder *Delta* with $\beta \to +\infty$ and other assumptions about the behavior of the vector for angular velocity ω.

2. Main results

Before formulating and proving the main results it is necessary to reconstruct the right part (10). First we must to obtain and estimate the integral with variable v, substituting distributions (5), (9) in (1)–(3) according to

$$D(M) = Q(M, M) = 0. \tag{11}$$

Thus,

$$D(f) = \int_{\mathbb{R}^3} D(\varphi) M \, du = \int_{\mathbb{R}^3} D(\varphi) e^{\beta\omega^2 r^2} \widetilde{M} \, du \tag{12}$$

$$\begin{aligned} Q(f,f) = \frac{d^2}{2} \int_{\mathbb{R}^3} dv_1 \int_{\Sigma} d\alpha |(v - v_1, \alpha)| \times \\ \times e^{2\beta\omega^2 r^2} \Bigg[\int_{\mathbb{R}^3} du_1 \varphi(t,x,u_1) \widetilde{M}(v_1', u_1, x) \int_{\mathbb{R}^3} du_2 \varphi(t,x,u_2) \widetilde{M}(v', u_2, x) - \\ - \int_{\mathbb{R}^3} du_1 \varphi(t,x,u_1) \widetilde{M}(v_1, u_1, x) \int_{\mathbb{R}^3} du_2 \varphi(t,x,u_2) \widetilde{M}(v, u_2, x) \Bigg], \end{aligned} \tag{13}$$

where the denotation

$$\widetilde{M} = \widetilde{M}(v,u,x) = \rho \left(\frac{\beta}{\pi} \right)^{\frac{3}{2}} e^{-\beta(v - \tilde{u})^2} \tag{14}$$

$$\tilde{u} = \tilde{u}(x) = u + [\omega \times x]$$

were introduced

Then

$$\begin{aligned} \int_{\mathbb{R}^3} |D(f) - Q(f,f)| \, dv = \int_{\mathbb{R}^3} \Bigg| \int_{\mathbb{R}^3} \left(\frac{\partial \varphi}{\partial t} + v \frac{\partial \varphi}{\partial x} \right) e^{\beta\omega^2 r^2} \widetilde{M} \, du - \\ - \frac{d^2}{2} \int_{\mathbb{R}^3} dv_1 \int_{\Sigma} d\alpha |(v - v_1, \alpha)| \times \\ \times e^{2\beta\omega^2 r^2} \int_{\mathbb{R}^6} \varphi(t,x,u_1) \varphi(t,x,u_2) \big[\widetilde{M}(v_1', u_1, x) \widetilde{M}(v', u_2, x) - \\ - \widetilde{M}(v_1, u_1, x) \widetilde{M}(v, u_2, x) \big] du_1 du_2 \Bigg| dv. \end{aligned} \tag{15}$$

As usual, we introduce the notion of "gain"G and "loss"L parts of the collision integral Q

$$G(f,g) = \frac{d^2}{2} \int_{\mathbb{R}^3} dv_1 \int_{\Sigma} d\alpha |(v - v_1, \alpha)| f(t, v_1', x) g(t, v_1, x), \tag{16}$$

$$L(g) = \frac{d^2}{2} \int_{\mathbb{R}^3} dv_1 \int_{\Sigma} d\alpha |(v - v_1, \alpha)| g(t, v_1, x). \tag{17}$$

Because, as is known [1]:

$$\int\limits_{\mathbb{R}^3} G(M_1, M_2)dv = \int\limits_{\mathbb{R}^3} M_1 L(M_2)dv, \tag{18}$$

we obtain the next upper estimation

$$\begin{aligned}\int\limits_{\mathbb{R}^3} |D(f) - Q(f,f)|\, dv \le \int\limits_{\mathbb{R}^6} \left|\frac{\partial\varphi}{\partial t} + v\frac{\partial\varphi}{\partial x}\right| e^{\beta\omega^2 r^2}\widetilde{M}\, dudv + \\ + e^{2\beta\omega^2 r^2}\int\limits_{\mathbb{R}^6} \varphi(t,x,u_1)\varphi(t,x,u_2)\int\limits_{\mathbb{R}^3}\big[\widetilde{M_1}L(\widetilde{M_2}) + \\ + \widetilde{M_2}L(\widetilde{M_1})\big]du_1du_2dv\end{aligned} \tag{19}$$

According to (19) for the correctly define of remainder (10) on coefficient functions it is necessary to impose new conditions of fast decrease on a spatial variable x. Therefore we will introduce the new denotation

$$\varphi(t,x,u) = \psi(t,x,u)e^{-\beta\omega^2 r^2}, \tag{20}$$

where the functions are continuously differentiable and non-negative. Then according to (19), (20) and (7) the estimate (19) takes a form

$$\begin{aligned}\int\limits_{\mathbb{R}^3} |D(f) - Q(f,f)|\, dv \le \\ \le \int\limits_{\mathbb{R}^6}\left|\frac{\partial\psi}{\partial t} + v\left(\frac{\partial\psi}{\partial x} - 2\beta\psi\big[[\omega\times(x-x_0)]\times\omega\big]\right)\right|\widetilde{M}\, dudv + \\ + \int\limits_{\mathbb{R}^6}\psi(t,x,u_1)\psi(t,x,u_2)\int\limits_{\mathbb{R}^3}\big[\widetilde{M_1}L(\widetilde{M_2}) + \widetilde{M_2}L(\widetilde{M_1})\big]du_1du_2dv\end{aligned} \tag{21}$$

Theorem. *Let conditions (5), (14), (20) be valid, and*

$$\omega = \frac{\omega_0 s}{\beta^k}, \tag{22}$$

where $s > 0$ is any constant, ω_0 is arbitrary fixed vector (the other parameters are also arbitrary and fixed so far). Also we assume that the functions ψ, $|\frac{\partial\psi}{\partial t}|$, $|\frac{\partial\psi}{\partial x}|$, $|[\omega_0\times x]|\psi$, $\left([\omega_0\times x], \frac{\partial\psi}{\partial x}\right)$, are bounded with respect to t and x on $\mathbb{R}^7$ and that the quantities

$$\psi,\ |u|\psi,\ \frac{\partial\psi}{\partial t},\ \frac{\partial\psi}{\partial x},\ u\frac{\partial\psi}{\partial x} \in L_1(\mathbb{R}^3) \tag{23}$$

in the variable u uniformly in $t$and x on $\mathbb{R}^4$. Then the quality Δ defined by formula (10) is meaningful (i.e., the finite inyegral and the finite supremum), and we have Δ' such that

$$\Delta \le \Delta'. \tag{24}$$

If

$$\frac{1}{2} < k \le 1, \tag{25}$$

or

$$\frac{1}{4} < k \le \frac{1}{2}, \tag{26}$$

and

$$[\omega_0 \times u] = 0, \tag{27}$$

then there is the finite limit

$$\begin{gathered} L = \lim_{\beta \to +\infty} \Delta' = \sup_{(t,x)\in\mathbb{R}^4} \left[\rho \int\limits_{\mathbb{R}^3} \left| \frac{\partial \psi}{\partial t} + u \frac{\partial \psi}{\partial x} \right| du + \right. \\ \left. + 2\pi^3 d^2 \rho^2 \int\limits_{\mathbb{R}^6} \psi(t,x,u_1)\psi(t,x,u_2)|u_1 - u_2| du_1 du_2 \right]. \end{gathered} \tag{28}$$

To prove Theorem we need the following lemma [7], which gives a sufficient condition for the continuity of supremum of the function of special kind of many variables. The supremum is taken respectively to a part of variables.

Lemma. *Let the function $g(y,z) : Y \times Z \to \mathbb{R}^1; Y \in \mathbb{R}^p; Z \in \mathbb{R}^q$; and let the following conditions be satisfied:*

1) $\forall z \in Z$, $g(y,z)$ is bounded in Y;

2) $g(y,z)$ is continuity in z uniformly with respect to y, i.e.,

$$\forall z_0 \in Z, \forall \varepsilon > 0, \exists \delta > 0, \forall y \in Y, \forall z \in Z,$$

$$|z - z_0| < \delta \Rightarrow |g(y,z) - g(y,z_0)| < \varepsilon$$

Then the function

$$l(z) = \sup_{y \in Y} |g(y,z)|$$

is continuity on the set Z.

Proof the Theorem. From (10), (21), (23) and the properties of supremum, there are follows the existence of the remainder Δ, and

$$\begin{gathered} \Delta \le \Delta' = \sup_{(t,x)\in\mathbb{R}^4} \left[\int\limits_{\mathbb{R}^6} \left| \frac{\partial \psi}{\partial t} + v \left(\frac{\partial \psi}{\partial x} - 2\beta\psi \big[[\omega \times (x - x_0)] \times \omega \big] \right) \right| \widetilde{M}\, dv du + \right. \\ \left. + \int\limits_{\mathbb{R}^6} \psi(t,x,u_1)\psi(t,x,u_2) \int\limits_{\mathbb{R}^3} \big[\widetilde{M_1} L(\widetilde{M_2}) + \widetilde{M_2} L(\widetilde{M_1}) \big] dv du_1 du_2 \right]. \end{gathered} \tag{29}$$

In (29) we also interchange the integration order; the validity of this procedure can be justified as follows.

The integrand in the first term is continuous and

$$\int\limits_{\mathbb{R}^6} \left| \frac{\partial \psi}{\partial t} + v \left(\frac{\partial \psi}{\partial x} - 2\beta\psi \big[[\omega \times (x - x_0)] \times \omega \big] \right) \right| \widetilde{M}\, du$$

converges uniformly in $\mathbb{R}^3$ (by the Weierstrass theorem),

$$\left| \left| \frac{\partial \psi}{\partial t} + v \left(\frac{\partial \psi}{\partial x} - 2\beta\psi \big[[\omega \times (x - x_0)] \times \omega \big] \right) \right| \rho \left(\frac{\beta}{\pi} \right)^{3/2} e^{-\beta(v - \tilde{u})^2} \right| \leq$$

$$\leq \rho \left(\frac{\beta}{\pi} \right)^{3/2} e^{-\beta(v - \tilde{u})^2} \left(\left| \frac{\partial \psi}{\partial t} \right| + |v| \left(\left| \frac{\partial \psi}{\partial x} \right| + 2\beta\psi \Big| [[\omega \times (x - x_0)] \times \omega] \Big| \right) \right)$$

is integrable by virtue of condition (23).

The integrand in the second term is continuous by the theorem conditions, and the inner integral converges uniformly in u_1 and u_2 by the Weierstrass theorem because we have an integrable majorizing function. We can therefore also change the integration order here.

Changing the variables as $\sqrt{\beta}(v - \widetilde{u}) = w$ and $v = \frac{w}{\sqrt{\beta}} + \tilde{u} = \frac{w}{\sqrt{\beta}} + u + [\omega \times x]$, we have

$$\Delta' = \sup_{(t,x) \in \mathbb{R}^4} \left[\rho \pi^{-3/2} \int\limits_{\mathbb{R}^6} \left| \frac{\partial \psi}{\partial t} + \left(\frac{w}{\sqrt{\beta}} + u + [\omega \times x] \right) \times \right. \right.$$
$$\left. \left. \times \left(\frac{\partial \psi}{\partial x} - 2\beta\psi \big[[\omega \times (x - x_0)] \times \omega \big] \right) \right| e^{-w^2} dw du \right] +$$
$$+ \sup_{(t,x) \in \mathbb{R}^4} \left[\int\limits_{\mathbb{R}^6} \psi(t, x, u_1)\psi(t, x, u_2) \int\limits_{\mathbb{R}^3} \left[\widetilde{M}_1 L(\widetilde{M}_2) + \widetilde{M}_2 L(\widetilde{M}_1) \right] dw du_1 du_2 \right]. \tag{30}$$

Let us consider the integrand of the second supremum in expression (30):

$$\widetilde{M}_1 L(\widetilde{M}_2) = \widetilde{M} \left(\frac{w}{\sqrt{\beta}} + \tilde{u}_1, u_1, x \right) \frac{d^2}{2} \int\limits_{\mathbb{R}^3} dv_1 \times$$
$$\times \int\limits_{\Sigma} d\alpha \left| \left(\frac{w}{\sqrt{\beta}} + \tilde{u}_1 - v_1, \alpha \right) \right| \widetilde{M}(v_1, u_2, x) = \widetilde{M} \left(\frac{w}{\sqrt{\beta}} + \tilde{u}_1, u_1, x \right) \frac{d^2}{2} \rho \left(\frac{\beta}{\pi} \right)^{3/2} \times$$
$$\times \int\limits_{\mathbb{R}^3} dv_1 \int\limits_{\Sigma} d\alpha \left| \left(\frac{w}{\sqrt{\beta}} + \tilde{u}_1 - v_1, \alpha \right) \right| e^{-\beta(v_1 - \tilde{u}_2)^2}.$$

We introduce the replacement

$$\sqrt{\beta}(v_1 - \tilde{u}_2) = z; \qquad v_1 = \frac{z}{\sqrt{\beta}} + \tilde{u}_2 = \frac{z}{\sqrt{\beta}} + u_2 + [\omega \times x] \,.$$

$$\widetilde{M}_1 L(\widetilde{M}_2) = \widetilde{M}\left(\frac{w}{\sqrt{\beta}} + \tilde{u}_1, u_1, x\right) \frac{d^2}{2} \rho \pi^{3/2} \int\limits_{\mathbb{R}^3} ds \int\limits_{\Sigma} d\alpha \left| \left(\frac{w}{\sqrt{\beta}} + \tilde{u}_1 - \frac{z}{\sqrt{\beta}} - \tilde{u}_2, \alpha\right)\right| e^{-z^2}.$$

Let θ be the angle between the vectors $\left(\frac{w}{\sqrt{\beta}} + \tilde{u}_1 - \frac{z}{\sqrt{\beta}} - \tilde{u}_2\right)$ and α. Then we have

$$\widetilde{M}_1 L(\widetilde{M}_2) = \widetilde{M}\left(\frac{w}{\sqrt{\beta}} + \tilde{u}_1, u_1, x\right) \frac{d^2}{2} \rho \pi^{3/2} \int\limits_{\mathbb{R}^3} dz e^{-z^2} \times$$

$$\times \int\limits_{\Sigma} d\alpha \left| \frac{w}{\sqrt{\beta}} + u_1 + [\omega \times x] - \frac{z}{\sqrt{\beta}} - u_2 - [\omega \times x] \right| |\cos\theta|.$$

We direct the z-axis along the vector $\left(\frac{w}{\sqrt{\beta}} + u_1 - \frac{z}{\sqrt{\beta}} - u_2\right)$ and introduce the spherical coordinate system on Σ. Integrating over the angles θ and φ, we then obviously obtain

$$M_1 L(M_2) = M\left(\frac{w}{\sqrt{\beta}} + u_1, u_1\right) \frac{d^2 \rho}{\sqrt{\pi}} \int\limits_{\mathbb{R}^3} dz e^{-z^2} \left| \frac{w}{\sqrt{\beta}} + u_1 - \frac{z}{\sqrt{\beta}} - u_2 \right|.$$

Analogously, we have

$$M_2 L(M_1) = M\left(\frac{w}{\sqrt{\beta}} + u_2, u_2\right) \frac{d^2 \rho}{\sqrt{\pi}} \int\limits_{\mathbb{R}^3} dz e^{-z^2} \left| \frac{w}{\sqrt{\beta}} + u_2 - \frac{w}{\sqrt{\beta}} - u_1 \right|.$$

Hence, we obtain

$$\int\limits_{\mathbb{R}^6} \psi_1 \psi_2 \int\limits_{\mathbb{R}^3} \left[M\left(\frac{w}{\sqrt{\beta}} + u_1, u_1\right) \beta^{3/2} \frac{d^2 \rho}{\sqrt{\pi}} \int\limits_{\mathbb{R}^3} dz e^{-z^2} \left| \frac{w}{\sqrt{\beta}} + u_1 - \frac{z}{\sqrt{\beta}} - u_2 \right| + \right.$$

$$\left. + M\left(\frac{w}{\sqrt{\beta}} + u_2, u_2\right) \beta^{3/2} \frac{d^2 \rho}{\sqrt{\pi}} \int\limits_{\mathbb{R}^3} dz e^{-z^2} \left| \frac{w}{\sqrt{\beta}} + u_2 - \frac{z}{\sqrt{\beta}} - u_1 \right| \right] dw du_1 du_2 =$$

$$= \int\limits_{\mathbb{R}^6} \psi_1 \psi_2 \int\limits_{\mathbb{R}^3} \left[\rho \pi^{-3/2} e^{-w^2} \frac{d^2 \rho}{\sqrt{\pi}} \int\limits_{\mathbb{R}^3} dz e^{-z^2} \left| \frac{w}{\sqrt{\beta}} + u_1 - \frac{z}{\sqrt{\beta}} - u_2 \right| + \right.$$

$$\left. + \rho \pi^{-3/2} e^{-w^2} \frac{d^2 \rho}{\sqrt{\pi}} \int\limits_{\mathbb{R}^3} dz e^{-z^2} \left| \frac{w}{\sqrt{\beta}} + u_2 - \frac{z}{\sqrt{\beta}} - u_1 \right| \right] dw du_1 du_2 =$$

$$= 2 \int\limits_{\mathbb{R}^6} \psi_1 \psi_2 \int\limits_{\mathbb{R}^3} \left[e^{-w^2} \frac{d^2 \rho^2}{\pi^2} \int\limits_{\mathbb{R}^3} dz e^{-z^2} \left| \frac{(w - z)}{\sqrt{\beta}} + u_1 - u_2 \right| \right] dw du_1 du_2$$

For expression simplification (30) we introduce the notation:

$$\gamma = \frac{1}{\sqrt{\beta}} \tag{31}$$

$$A(w,u,t,x) = \rho \frac{d^2}{\sqrt{\pi}} \int\limits_{\mathbb{R}^3} dz e^{w^2} \left| w\gamma + (u_1 - u_2) - z\gamma \right| \tag{32}$$

$$\begin{aligned} B(w,u,t,x) &= \left(\frac{w}{\sqrt{\beta}} + u + [\omega \times x] \right) \left(\frac{\partial \psi}{\partial x} - 2\beta\psi \big[[\omega \times (x - x_0)] \times \omega \big] \right) = \\ &= \left(\frac{w}{\sqrt{\beta}} + u + [\omega \times x] \right) \times \\ &\times \frac{\partial \psi}{\partial x} + 2\beta\psi \left(\omega\left(\omega, x - x_0\right) - \omega^2 \left(x - \frac{[\omega \times u]}{\omega^2} \right) \right) = \\ &= \left(\frac{w}{\sqrt{\beta}} + u + [\omega \times x] \right) \times \frac{\partial \psi}{\partial x} + 2\beta\psi \big(\omega\left(\omega, x\right) - x\omega^2 + [\omega \times u] \big) = \\ &= \frac{\partial \psi}{\partial x} \left(\frac{w}{\sqrt{\beta}} + u + [\omega \times x] \right) + \\ &+ 2\psi\sqrt{\beta} \left\{ (\omega, w)(\omega, x) - \omega^2 (x, w) + (w, [\omega \times u]) \right\} = \\ &= \frac{\partial \psi}{\partial x} \left(\frac{w}{\sqrt{\beta}} + u + [\omega \times x] \right) + \\ &+ 2\psi\sqrt{\beta} \left\{ -[\omega \times w][\omega \times x] + (w, [\omega \times u]) \right\} \end{aligned} \tag{33}$$

Using expressions (23) and (31), we will obtain the following:

$$\begin{aligned} B(w,u,t,x) = \frac{\partial \psi}{\partial x} \left(w\gamma + u + \gamma^2 s[\omega_0 \times x] \right) + \\ + 2\psi\gamma s \left\{ (w, [\omega_0 \times u]) - s\gamma^2 [\omega_0 \times w][\omega_0 \times x] \right\} \end{aligned} \tag{34}$$

Taking formulas (32) and (34) into account, we rewrite expression (30) as

$$\begin{aligned} \Delta' = \sup_{(t,x) \in \mathbb{R}^4} \left[\rho\pi^{-3/2} \int\limits_{\mathbb{R}^3} \left| \frac{\partial \psi}{\partial x} + B(w,u,t,x) \right| e^{w^2} dw du \right] + \\ + \sup_{(t,x) \in \mathbb{R}^4} \left[2\rho\pi^{-3/2} \int\limits_{\mathbb{R}^6} \psi(t,x,u_1)\psi(t,x,u_2) \int\limits_{\mathbb{R}^3} A(w,u_1,u_2,t,x) dw du_1 du_2 \right] \end{aligned} \tag{35}$$

We apply the aforesaid Lemma to each supremum contained in (35), where $y = (t,x), Y = \mathbb{R}^4, z = (w,\gamma),\ Z = \mathbb{R}^3 \times \mathbb{R}^1_+$. Fulfillment of the first and second conditions follows from (23), (32), (34) and the theorem conditions.

Because the lemma conditions are satisfied for each of these supremums, the whole quantity Δ' is continuous in γ on $\mathbb{R}^1_+$. So, in (35) we can pass to the limit with $\beta \to +\infty$, which is equivalent to the tending of γ to zero. Thus dependence on z and w is reduced only to expressions e^{z^2} in (32) and e^{w^2} in (35). As a result of integration by z and w we come to (28).

Now, based on the obtained expression for the limit as $\beta \to +\infty$, we can find the sufficient condition for the mismatch Δ to tend to zero, which we formulate as a corollary of Theorem.

Corollary. *Let all the theorem conditions be valid. Then the statement*

$$\Delta \to 0 \tag{36}$$

holds if the function ψ defined by formula (8) has the form

$$\psi(t, x, u) = C(x - ut) \left(\frac{P}{\pi}\right)^{3/2} e^{-P(u-u_0)^2}, \tag{37}$$

where C is any smooth, positive and bounded function together with all its derivatives, $u_0 \in \mathbb{R}^3$ is an arbitrary fixed vector, and $P \to +\infty$.

Proof. Let us use limit expression (28) and substitute expression (37) in it. The integrand of the first term then vanishes,

$$C'(x - ut)(-u) + uC'(x - ut) = 0.$$

We consider the integral in the second term (the proof of integral convergence and it tending to zero as $P \to +\infty$ is analogous to proof in [12]). The corollary is proved.

Remark 1. Relation (36) also holds at a fixed P in expression (37) under the additional condition that $d \to 0$ (the near-Knudsen gas).

Remark 2. In (37), we can obviously take $C\big([u \times x]\big)$ instead of the first factor $C(x - ut)$ and take other δ-functions instead of the second factor.

Remark 3. The common property of all obtained distributions is that they describe the non-uniform cooling gas ($\beta \to +\infty$). Besides, the rotation of spiral decelerates ($\omega \to 0$), although in different degrees in accordance with (22) and under the conditions of (25), (26). As corollary shows, the estimate (21) and the limit in expression (28) ensure the further arbitrary smallness of the remainder Δ for the given coefficient functions and sufficiently small absolute temperature, which only means assuming that the thermal constituent of the molecule velocities is small when an arbitrary value of the mass velocity of a flow is preserved. At the same time, the distribution f itself does not tend to any of Maxwellians (i.e. to the known exact solution of the Boltzmann equations). It's defined by (37), (9), (5).

In summary, in this paper we managed a few to generalize results, which obtained in [12] and [14]. We have here constructed continual distribution for the case of local Maxwellians describing the screw-shaped stationary equilibrium states of a gas and satisfying Boltzmann equation (1)–(3) in the sense of minimizing mismatch (10).

REFERENCES

1. Cercignani C. Theory and Application of the Boltzmann Equation. – Scottish Academic Press, Edinburgh, 1975.

2. Kogan M. Dynamics of Diluted Gas. – Nauka, Moscow, 1967. (in Russian).

3. Carleman T. Problems Mathematiques dans la Theorie Cinetique des Gas. – Almqvist Wiksells, Uppsala, 1957.

4. Bobylev A.V. Exact solutions of the Boltzmann equation. // Sov. Phys. Dokl., 1976. – **20**. – P. 822–824.

5. Krook K., Wu T.T. Exact solutions of the Boltzmann equation // Phys. Fluids, 1977. – V. 20, **10(1)**. – P. 1589–1595.

6. Ernst H.M. Exact solutions of the nonlinear Boltzmann equation // J. Statist. Phys., 1984. – V. 34, **5/6**. – P. 1001–1017.

7. Gordevskyy V.D. Biflow Distributions with Screw Modes // Theor. Math. Phys., 2001. – V. 126, **2**. – P. 234–249.

8. Gordevskyy V.D. Rotating Flows with Acceleration and Compacting in the Model of Hard Spheres // Theor. Math. Phys., 2009. – V. 161, **2**. – P. 1558–1566.

9. Gordevskyy V.D. Transitional regime between vortical states of a gas // Nonl. Analysis (NA 3752), 2003. – V. 53, **3-4**. – P. 481–494.

10. Gordevskyy V.D. Vorticies in a gas of hard spheres // Theor. Math. Phys., 2003. – V. 135, **2**. – P. 704–713.

11. Gordevskyy V.D. Trimodal Approximate Solutions of the Non-linear Boltzmann Equation // Math. Meth. Appl. Sci., 1998. – **21**. – P. 1479–1494.

12. Gordevskyy V.D., Sazonova E.S. Continuum analogue of bimodal distribution // Theor. Math. Phys., 2012. – V. 171, **3**. – P. 839–847.

13. Gordevskyy V.D. An approximate two-flow solution to the Boltzmann equation // Theor. Math. Phys., 1998. – V. 114, **1**. – P. 99–108.

14. Gordevskyy V. D., Sazonova E. S. Asymmetrical bimodal distributions with screw modes // Math. Phys., Anal., Geom., 2011. – V. 7, **3**. – P. 212–224.

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